NUMERICAL INVESTIGATION OF THE STABILITY OF THE EQUILIBRIUM OF A CYLINDRICAL LAYER OF FLUID WITH INTERNAL HEAT SOURCES

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In the absence of volume forces, the temperature dependence of the coefficient of surface tension and the resultant thermocapillary effect are the primary influences on the stability of the equilibrium of a non-uniformly heated fluid. If the equilibrium temperature gradient is sufficiently large, then the presence of thermocapillary forces at the free surface can lead to the onset of convective motion.

Investigation of thermocapillary equilibrium instability with respect to monotonic perturbations was carried out in [1-4]. Although an attempt was made to study non-monotonic perturbations in a plane layer [5, 6], no such analysis was done for a cylindrical region. At the same time, it is known [7, 8] that the presence of the capillary mechanism of convection in the case of a cylinder gives rise to oscillatory perturbations which are neutrally stable. In this work, stability with respect to arbitrary perturbations of the equilibrium of a cylindrical layer is studied. The present numerical analysis of monotonic instability shows that for a deformable free surface of neutral curvature, there is not one curve with a point of discontinuity, but two independent curves. Each of these corresponds to its own type of perturbation: capillary or thermocapillary. In addition, when considering the capillary convection mechanism, there is an oscillatory instability, which occurs for axisymmetric and azimuthal (m = 1) perturbations. Oscillatory perturbations are stabilized by capillarity for other azimuthal modes. The presence of the thermocapillary mechanism was observed to stabilize Rayleigh instabilities in the long-wavelength region.

1. We consider a cylindrical layer of viscous, heat-conducting fluid bounded by rigid internal and free external surfaces, in the absence of volume forces. We introduce a cylindrical coordinate system with the z axis directed along the generator of the cylinder. The equations of the rigid and free boundaries are $r = r_0$ and $r = r_1$, respectively. The temperature dependence of the coefficient of surface tension is given by $\sigma = \sigma_0 - \kappa\Theta$.

Let heat sources of intensity q = const be uniformly distributed in the fluid. Then the equilibrium state is described by

$$u = v = w = 0, \quad p = \text{const}, \quad \Theta(r) = q\left[2r_0^2 \ln(r/r_1) - r^2 + r_1^2\right]/4\chi. \tag{1.1}$$

Here (u, v, w) are the components of the vector velocity; p is the pressure; Θ is the temperature; and χ the thermal diffusivity coefficient.

We choose as units of length, time, velocity, pressure and temperature r_1, r_1^2/ν, ν/r_1, $\rho\nu^2/r_1^2$, $\nu\gamma r_1/\chi$, respectively. Here ρ is the density; ν the fluid kinematic viscosity coefficient; and $\gamma = qr_1/2\chi$. Then the expression for the equilibrium temperature takes the form

$$\Theta_0(\xi) = \mathrm{Pr}^{-1}(2d^2 \ln \xi + 1 - \xi^2).$$

We seek perturbations to the vector velocity, pressure, temperature, and normal component of the free surface in the form

$$(U, V, W, P, T, R) = (U(\xi), V(\xi), W(\xi), P(\xi), T(\xi), R) \times \exp[i\alpha\eta + im\varphi - i\alpha C\tau],$$

where α, m are the axial and azimuthal wavenumbers; $C = C_r + iC_i$ is the complex decrement; and τ is dimensionless time. The sign of the imaginary part of the decrement serves as the criterion for the stability of the equilibrium state of (1.1). Values of problem parameters

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for which $C_i < 0$ correspond to regions of stability; if there also exist values of the parameters for which $C_i > 0$, then we have instability. The case $C_i = 0$ corresponds to the boundary of stability (neutral perturbations).

The equations for small perturbations take the form [9]

$$aU + P' = -i\alpha W' - \frac{im}{\xi^2}(\xi V)'',$$
$$aV + \frac{im}{\xi}P = \left[\frac{1}{\xi}(\xi V)'\right]' + \frac{2im}{\xi^2}U, \tag{1.2}$$
$$aW + i\alpha P = \frac{1}{\xi}(\xi W')', \quad \frac{1}{\xi}(\xi U)' + \frac{im}{\xi}V + i\alpha W = 0,$$
$$bT - \left(\frac{d^2}{\xi} - \xi\right)U = \frac{1}{\xi}(\xi T')' \quad (d < \xi < 1);$$
$$a = -i\alpha C + \alpha^2 + m^2/\xi^2, \quad b = -i\alpha\,\mathrm{Pr}\,C + \alpha^2 + m^2/\xi^2;$$

the conditions at the rigid boundary ($\xi = d$) are:

$$U = V = W = T' = 0; \tag{1.3}$$

at the free surface ($\xi = 1$):

$$V' - V + imU = -im\mathrm{M}(T + \Theta_0' R); \tag{1.4}$$
$$i\alpha U + W' = -i\alpha\mathrm{M}(T + \Theta_0' R); \tag{1.5}$$
$$-i\alpha CR = U; \tag{1.6}$$
$$-P + 2U' = -\mathrm{M}(T + \Theta_0' R) + \mathrm{We}(1 - \alpha^2 - m^2)R; \tag{1.7}$$
$$T' + \mathrm{Bi}\,T + (\Theta_0'' + \mathrm{Bi}\,\Theta_0')R = 0. \tag{1.8}$$

Here $\xi = r/r_1$; $d = r_0/r_1$; $\mathrm{M} = \gamma\kappa r_1^2/\rho\nu\chi$ is the Marangoni number; $\mathrm{We} = r_1\sigma_0/\rho\nu^2$ is the Weber number; $\mathrm{Pr} = \nu/\chi$ the Prandtl number; $\mathrm{Bi} = \beta r_1/\lambda$ is the Biot number; and λ, β are the coefficients of thermal conductivity and interphase heat exchange.

In order to study the thermocapillary mechanism of convection independently of the consideration of capillarity, it is useful to consider the problem in a simplified situation (as was done, for example, in [1, 3]); namely, to consider that the free surface is undeformed ($R = 0$). In this case, the instability can arise only because of nonuniformities in the temperature distribution at the free surface. In our problem, this condition, except for the cases $\alpha = 1$, $m = 0$ and $\alpha = 0$, $m = 1$ can formally be obtained from (1.7) by setting $\mathrm{We} = \infty$. Then the boundary conditions for $\xi = 1$ take the form

$$V' - V + im\mathrm{M}T = 0, \quad W' + i\alpha\mathrm{M}T = 0, \quad U = 0, \tag{1.9}$$
$$T' + \mathrm{Bi}\,T = 0.$$

For exclusions listed above, for $m = 0$, the line $\alpha = 1$ corresponds to the Rayleigh instability boundary [9] and for the thermocapillary mechanism studied here, is not considered. For $\alpha = 0$, $m = 1$ only the trivial solution holds [4].

2. We carry out an asymptotic analysis of the long-wavelength perturbations ($\alpha \to 0$) for $m = 0$. In this case, the problem for the function V is separate. Letting $\mathrm{We} = \infty$, and setting $\alpha C = O(\alpha)$, we obtain the characteristic equation governing the complex decrement:

$$\gamma[J_1(\gamma)Y_1(\gamma d) - Y_1(\gamma)J_1(\gamma d)] + \mathrm{Bi}[Y_0(\gamma)J_1(\gamma d) - J_0(\gamma)Y_1(\gamma d)] = 0 \tag{2.1}$$

($\gamma = (i\alpha C\mathrm{Pr})^{1/2}$; J_0, J_1, Y_0, Y_1 are Bessel functions of first and second kind). Equation (2.1) has a countable number of real roots.

If $\mathrm{We} \neq \infty$, then assuming that $C = O(\alpha)$, $T = O(1/\alpha)$ for $\alpha \to 0$, $\mathrm{Bi} \neq 0$, we find

$$C = -\frac{i\alpha}{4}\left\{\mathrm{We}\left[\frac{(1-d^2)^2}{4} + \frac{1-d^2}{2} + \ln d\right] - \mathrm{M}(1+d^2)\left[\frac{(1-d^2)^2}{4} + \frac{3(1-d^2)}{2} + 3\ln d\right]\right\}. \tag{2.2}$$

Analysis of (2.1) and (2.2) shows that for $m = 0$, $\mathrm{Bi} \neq 0$, long-wavelength perturbations are always monotonic, and for $\mathrm{We} = \infty$, the equilibrium state is stable.

The solution of (1.2)-(1.8) for monotonic, neutral perturbations ($C = 0$) is constructed in [4]. The analytic dependence obtained there of the Marangoni number on the remaining parameters was used as a check on the calculations.

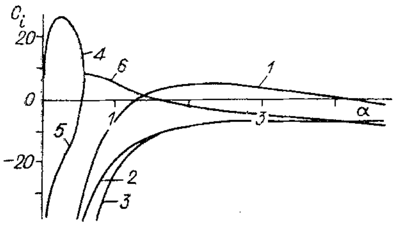

Fig. 1

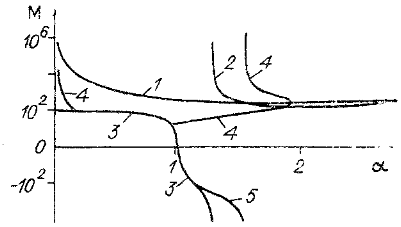

Fig. 2

3. Numerical solution of (1.2)-(1.8) was carried out by using the method of orthogonalization. The asymptotic values found from (2.1), (2.2) were used as starting approximations. The effect of capillary and thermocapillary mechanisms on the stability of equilibrium was studied in the case of a melt of germanium with Pr = 0.016, Bi = 2. We consider an axisymmetric perturbation (m = 0), setting d = 0.1.

Let the free surface be undeformable (We = ∞). The calculations show that in this case, the loss of stability occurs at M > 216, which coincides with the results of [4]. Moreover, for all α, it was found that $C_r = 0$. Figure 1 shows the results of calculations of C_i versus α for the most dangerous modes, constructed for M = 300 and 80 (curves 1 and 2). The corresponding neutral curve is shown in Fig. 2 (curve 1). Thus analysis of the numerical and analytical results shows that in the case of a nondeformable free surface (the thermocapillary mechanism arises by convection), only monotonic perturbations are realized.

Taking the deformation of the free surface into account leads to equilibrium destabilization. In this case, the spectrum of the most dangerous perturbations has a more complex form, shown in Fig. 1 for M = 80. Curve 3 illustrates the change in the thermocapillary mode with decreasing Weber number. Note that C_r is always equal to zero for this mode, independent of We. The neutral curve for We = 10^4, which corresponds to this perturbation, is shown in Fig. 2 (curve 2). In addition, accounting for the deformation of the free surface leads to the appearance of a new mechanism of instability. As shown in Fig. 1, two new modes appear in this case, which are monotonic in the region of small α (curves 4, 5). Here curve 4 lies in the upper half plane and begins with the asymptote (2.2). The intersection points of curve 5 with the axis $C_i = 0$ (α = 0.53 for M = 80) form a monotonic capillary neutral curve, which is shown in Fig. 2 (curve 3) for We = 10^4. With increasing Marangoni number, curves 4 and 5 are joined from below and for 94.5 < M < 95.6, the monotonic capillary curve is determined by the intersection points of curve 4 with the axis $C_i = 0$. For M > 95.6, the monotonic instability in the region of small α disappears.

Let us examine the onset of oscillatory instability. As shown in Fig. 1, with increasing α, the monotonic capillary modes merge, forming a complex-conjugate pair. The imaginary part of the decrement in Fig. 1 corresponds to curve 6. The neutral curve for oscillatory instability is shown in Fig. 2 (curve 4). It consists of two parts connected together by curve 3; the transition points from one curve to the other are: α = 0.16, M = 94.5, and α = 0.95, M = 18.9.

In addition to heating of the fluid, an analogous mechanism for the onset of oscillatory instability pertains during coolong as well. In this case, as shown in Fig. 2, the neutral curve of oscillatory perturbations (curve 5) branches from the neutral curve of capillary monotonic perturbations. The stability region in Fig. 2 is bounded on the left and from below by curve 3 for 0.95 < α < 1.14, and by curve 5 for 1.14 < α < 1.56. The boundary continues from above along curve 4 for 0.95 < α < 1.92 and along curve 2 for α > 1.92. There is in addition, an "islet of stability" in the small wavenumber region, bounded from above by the left part of curve 4 and from below by the straight line M = 95.6. Note that curve 5 and the upper branch of curve 4 have the same asymptote α = 1.56, and consequently, similar behavior of the neutral curves is observed for both monotonic and oscillatory perturbations.

Thus all perturbations are divided into two types: thermal and capillary, each of which has its own instability mechanism. The thermal instability is caused by the presence of temperature nonuniformities at the free surface and the associated thermocapillary effect. Perturbations of this type are always monotonic and play a leading role at large wavenumbers. This instability mechanism was first described in [1].

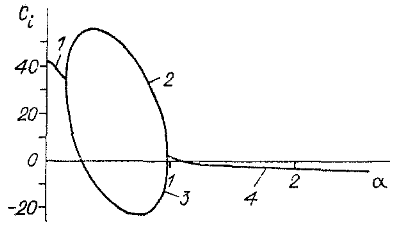

Fig. 3

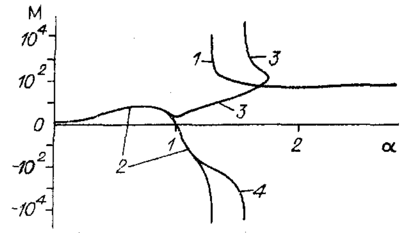

Fig. 4

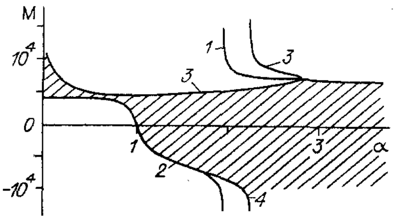

Fig. 5

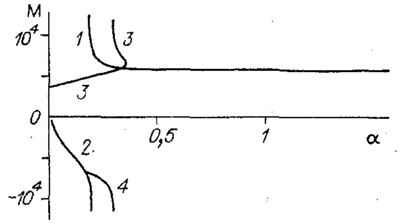

Fig. 6

Capillary perturbations arise due to the deformability of the free boundary and take the form of a pair of waves moving along the surface. This type of instability dominates at small α and was first investigated by Rayleigh [7]. The simultaneous effect of both mechanisms leads to the onset of oscillatory perturbations, with respect to which the instability is possible only in a bounded wavenumber interval ($\alpha < 1.95$ for $We = 10^4$). In this case, the short-wavelength perturbations are stabilized by surface tension forces. In addition, the presence of the thermocapillary convection mechanism stabilizes the Rayleigh instability in the long-wavelength region.

The above description of the most dangerous perturbations is characteristic of nonvanishing Biot numbers. If the free surface is thermally insulated ($Bi = 0$), then there will be another mechanism for the onset of oscillatory perturbations. Figure 3 shows a plot of capillary modes constructed for $Bi = 0$, $M = 2$, and $We = 10^4$. It is evident that the long-wavelength perturbations are always oscillatory and that the equilibrium state is unstable with respect to these perturbations. With decreasing wavelength, the oscillatory mode (curve 1) decays to two monotonic branches (curves 2, 3), the lower of which intersects the axis $C_i = 0$ and forms a monotonic neutral curve. With subsequent increase in wavenumber, the monotonic modes merge, once again forming a complex-conjugate pair (curve 4). The corresponding neutral curves are shown in Fig. 4. Here, curves 1 and 2 are due to monotonic perturbations, and curves 3, 4 to oscillatory perturbations. The region of stability is simply connected and is bounded on the left by curves 2 and 3.

Let us examine the influence of other parameters of this problem on the behavior of the neutral curves. An interesting effect was observed with decreasing dimensionless layer thickness, namely: with growth in d, the transition points of the oscillatory neutral curve to the monotonic curve are shifted towards one another and for a some d_0, a "bridge" appears between the regions of stability. As an illustration, we examine Fig. 5, which shows the neutral curves constructed for $d = 0.5$, $Bi = 2$, and $We = 10^4$ (for these values of Bi and We, we obtain $d_0 = 0.447$). The region of stability is shaded. Similar behavior of the neutral curves is also observed for decreasing Pr. In this case, the formation of the connected region of stability takes place at very small values of Pr (on the order of 10^{-4}). In addition, decreases in Pr lead to destabilization of equilibrium.

Let us examine azimuthal perturbations ($m \neq 0$). It is known [8] that in the presence of the capillary mechanism of convection, these perturbations are always stable. However, taking thermocapillarity into account leads to the appearance of an oscillatory instability, which can be the most dangerous. Figure 6 shows the neutral curves constructed for $m = 1$, $Bi = 2$, $Pr = 0.016$, and $We = 10^4$ (curves 1, 2 correspond to monotonic, and curves 3, 4 to oscillatory perturbations). Thus, there is a qualitative coincidence of behavior of the azimuthal and axisymmetric neutral curves. In this case, the axisymmetric perturbations will be more dangerous than the azimuthal perturbations. The numerical analysis given here shows that oscillatory instability is possible only for $m = 1$. With increasing azimuthal wavenumber, the oscillatory perturbations are stabilized by capillarity.

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CALCULATION OF THE FLOW FIELD PAST SPHERICALLY BLUNTED CONES NEAR THE PLANE OF SYMMETRY FOR VARIOUS SHOCK LAYER FLOW REGIMES WITH INSUFFLATION OF GAS FROM THE SURFACE

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We investigated flow past a cone, which has been blunted in a spherical fashion, over a wide range of Reynolds numbers. Various flow regimes were realized in the shock layer. The study was done within the framework of a completely viscous shock layer model, near the plane of symmetry of the flow. In the shock layer for this flow, the problem of self-consistent calculation for the plane of symmetry was treated in [1, 2] by means of a Fourier series expansion of the pressure in terms of the circumferential coordinate. In [3], a prescribed pressure gradient in the circumferential coordinate taken from tables of inviscid flow was used to model a thin viscous shock layer. Here, we apply the truncated series procedure [4], and analyze the effect of the angle of attack α_A and taper angle β on the heat exchange characteristics. The case where α_A is significantly larger than β is included in our analysis. We also analyze the effect of discharge quantity and the distribution law of gas insufflating through a porous, spherical shell on the heat exchange characteristics.

1. Let us write out the system of equations for a viscous shock layer in the neighborhood of the flow plane of symmetry in the natural coordinate system (s, ψ, n), attached to the body axis of symmetry. Using an expansion of the coefficients and unknown functions of the form

$$f = f_0 + f_2\psi^2 + \ldots \ (f = u,\ v,\ H,\ \rho,\ \mu,\ p,\ h,\ n_s),$$
$$\omega = \omega_1\psi + \ldots$$

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